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ORIGINAL ARTICLE

Approximate formulas for rotational effects in earthquake engineering

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Abstract The paper addresses the issue of researching into the engineering characteristics of rotational strong ground motion components and rotational effects in structural response. In this regard, at first, the acceleration response spectra of rotational components are estimated in terms of translational ones. Next, new methods in order to consider the effects of rotational components in seismic design codes are presented by determining the effective structural parameters in the rotational loading of structures due only to the earthquake rotational components. Numerical results show that according to the frequency content of rotational components, the contribution of the rocking components to the seismic excitation of short period structures can never be ignored. During strong earthquakes, these rotational motions may lead to the unexpected overturning or local structural damages for the low-rise multi-story buildings located on soft soil. The arrangement of lateral-load resisting system in the plan, period, and aspect ratio of the system can severely change the seismic loading of wide symmetric buildings under the earthquake torsional component.

Keywords Earthquake rotational components · Apparent velocity · Principal axes · Acceleration response spectrum · Base shear · Accidental eccentricity

1 Introduction

The study on the seismic behavior of the engineering structures during the past strong ground motion (SGM) has shown that parts of earthquake damages or even collapse of structures cannot only be attributed to the translational components of SGMs. Indeed, some unexpected failures of structures such as tall asymmetric buildings or irregular frames (Ghafory-Ashtiany and Singh 1984), bridges (Kalkan and Grazer 2007), slender tower-shaped structures (Zembaty and Boffi 1994), nuclear reactors (Rutenberg and Heidebrecht 1985), vertically irregular buildings (Ghafory-Ashtiany and Falamarz-Sheikhabadi 2010), and even ordinary multi-story buildings near earthquake faults (Trifunac 2009), can be associated with the seismic loading due to spatial variation of seismic waves. The spatial derivatives of translational SGMs are named rotational components of the SGM and their influences on the seismic behavior of structures have been the subject of many theoretical researches during the past 40 years (Newmark 1969;

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Trifunac 1982; Ghafory-Ashtiany and Singh 1986; Twiss et al. 1993; De La Llera and Chopra 1994; Hao 1996; Shakib and Tohidi 2002; Li et al. 2004; Ghayamghamian and Nouri 2007; Pujol 2009; Lee and Trifunac 2009). However, due to the lack of the recorded data on the rotational components, the earthquake-resistant design of structures are mainly performed by considering the influences of the translational components and the seismic loading due to the rotational components are ignored or underestimated by most seismic codes.

So far, the effects of the rocking components on the seismic loading of structures are only regarded by Eurocode 8, part 6 (EC8.6, 2005) which recommends that the rocking seismic excitations should be considered for the tall structures (higher than 80 m) designed in regions of high seismicity. The response spectrum for the rocking component is defined as (Eurocode 8 et al. 2005):

$$\mathrm{SA}_{\theta y}(T) = \frac{1.7\pi \mathrm{SA}_u(T)}{V_S T} \tag{1.a}$$

where $\mathrm{SA}_u(T)$ is the elastic horizontal response spectrum defined for the site classes, based on the average shear wave (S-wave) velocity over the uppermost 30 m of the ground profile, V_s, and T is the natural period of structure. Formula 1.a represents only rocking excitations and does not take into account soil-structure interaction effects, which may excite structural rocking even only from the horizontal SGM. This code also presents a formula similar to Eq. 1.a to consider the effects of the torsional component:

$$\mathrm{SA}_{\theta z}(T) = \frac{\pi \mathrm{SA}_u(T)}{V_S T} \tag{1.b}$$

However, in most of the seismic codes, the effects of torsion are usually considered in the case of the structural irregularities. This can be done by applying the equivalent lateral forces at a distance e_d (design eccentricity) from the center of rigidity (CR). Some codes also specify the design eccentricity with respect to the shear center. The code provision for the design eccentricity at the fth floor, e_{df}, can be expressed in a general form as:

$$e_{\mathrm{df}} = \alpha e_f + \beta b_f \tag{2.a}$$

$$e_{df} = \delta e_f + \beta b_f \tag{2.b}$$

where e_f is the static eccentricity at the fth floor defined between the floor center of mass (CM) and the CR, b_f is the plan dimension of the fth floor normal to the considered direction of ground motion, and coefficients α, β, and δ are the code-specified constants. The second term in Eqs. 2.a and 2.b is introduced in codes to account for differences between the analytical and actual location of centers of mass, shear, and resistance in structures during an SGM. This accidental eccentricity is assumed to be a fraction of the plan dimension, βb_f, where the coefficient β based on the finding of the elastic analysis of rigidity supported structures and on engineering judgment is proposed to be in the range of 0.05–0.1 in most of the seismic design codes.

Past studies have shown that Eqs. 1.a and 1.b for the rotational excitations require further research and empirical scaling (Ghayamghamian et al. 2009). The same is true when it comes to the formulas for structural eccentricities (Zembaty 2009).

In this study, methods for inclusion of the loading effects of earthquake rotational components in building codes are proposed. To achieve this, at first, the characteristics of rotational components and their relations with corresponding translational components are reviewed. Next, the effective structural parameters and their influences on the rotational excitation of structures subjected to the earthquake rotational components are determined. Besides, the random responses of the structures under the combined action of the translational and rotational components are analyzed. Finally, based on the obtained results of dynamic analyses, the simple formulas for the estimation of the seismic loading of structures due to the earthquake rotational components are presented.

2 Characteristics of rotational components

The rotational components of ground motions, $\left\{\vec{\theta}^g\right\}$, induced by the spatial variation of seismic waves can be obtained in terms of the translational components, $\left(u_x^g, u_y^g, u_z^g\right)$, along Cartesian coordinates axes (x, y, z) for small deformation as follows. Defining Cartesian coordinate system on the ground surface $(z = 0)$, displacement gradient, ∇U, which is a second tensor order, will become:

$$\nabla U = \begin{bmatrix} \frac{\partial u_x^g}{\partial x} & \frac{1}{2}\left(\frac{\partial u_y^g}{\partial x} + \frac{\partial u_x^g}{\partial y}\right) & 0 \\ \frac{1}{2}\left(\frac{\partial u_y^g}{\partial x} + \frac{\partial u_x^g}{\partial y}\right) & \frac{\partial u_y^g}{\partial y} & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3} + \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{\partial u_y^g}{\partial x} - \frac{\partial u_x^g}{\partial y}\right) & -\frac{\partial u_z^g}{\partial x} \\ \frac{1}{2}\left(\frac{\partial u_y^g}{\partial x} - \frac{\partial u_x^g}{\partial y}\right) & 0 & -\frac{\partial u_z^g}{\partial y} \\ \frac{\partial u_z^g}{\partial x} & \frac{\partial u_z^g}{\partial y} & 0 \end{bmatrix}_{3\times 3} \tag{3}$$

where the symmetric matrix corresponds with the strain tensor in small deformation and the anti-symmetric matrix is the rotation tensor. From this equation, the rotational components vector, $\vec{\theta}$, can be expressed as:

$$\vec{\theta} = \frac{\partial u_z}{\partial y}\vec{i} - \frac{\partial u_z}{\partial x}\vec{j} + \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)\vec{k} \tag{4}$$

The first two terms in right side of Eq. 4 are known as the rocking components related to the vertical ground motion, and third term entitled the torsional component is related to the horizontal motions. Transforming Eq. 4 into the frequency domain, the rotational displacements corresponding to the linear SGMs can be obtained as:

$$\left\{\vec{\theta}^g(\omega)\right\} = 2\pi i\frac{u_z^g(\omega)}{\iota_y(\omega)}\vec{i} - 2\pi i\frac{u_z^g(\omega)}{\iota_x(\omega)}\vec{j} - \pi i\left(\frac{u_y^g(\omega)}{\iota_x(\omega)} - \frac{u_x^g(\omega)}{\iota_y(\omega)}\right)\vec{k} \tag{5}$$

where ω is the circular frequency, ι_j is the wavelength of seismic waves along jth-direction on horizontal surface, and $i=\sqrt{-1}$. By introducing the equivalent constant apparent velocities for x- and y-directions, as the velocity at which a plane wave appears to travel along horizontal surface, the rotational acceleration components of the linear earthquake ground motions can be estimated as:

$$\left\{\ddot{\vec{\theta}}^g(\omega)\right\} = \left\{i\omega\frac{\ddot{u}_z^g(\omega)}{V_y}; -i\omega\frac{\ddot{u}_z^g(\omega)}{V_x}; i\frac{\omega}{2}\left(\frac{\ddot{u}_x^g(\omega)}{V_y} - \frac{\ddot{u}_x^g(\omega)}{V_x}\right)\right\} \tag{6}$$

where V_j is the constant apparent velocity along jth-direction. This simple relation can be used as a first-order approximation in calculating the acceleration response spectra of rotational components of SGMs with an acceptable approximation except in the highly attenuated medium. For a homogeneous isotropic and elastic semi-infinite medium, the apparent velocity in Eq. 6 can be assumed as $V_{S-S}/\sin\phi$, which $V_{s\text{-}s}$ is the propagation velocity of the shear waves in the medium and ϕ is the incident angle. Thus, in this case, the theoretical value of the apparent velocity is in the range of the propagation velocity of the shear waves in the medium and infinity. From engineering aspect, considering the fact that the soil beneath each structure is usually assumed to be horizontally layered, in most of the seismic codes, a constant velocity equal to the S-wave velocity over the uppermost 30 m of the ground profile is considered as the apparent velocity of seismic waves. It should be kept in mind that such an assumption usually gives the most conservative form of the rotational components unless some special situations which site effects cause seismic waves propagate horizontally. Here, it should also be mentioned that based on the authors' knowledge, there is not any exact theoretical method to estimate rotational components in near field and we can only estimate rotational motions in the far distances from fault zone, appropriately. In fact, for the estimation of the rotational components in near field, we have infinity solutions for solving seismic wave propagation problem. Using shear wave velocity at surface layer instead of apparent velocity in seismic codes is probably due to this difficulty in defining and estimating phase velocity. Since authors have recently improved the concept of bidirectional phase velocity in the middle-field zone, to present engineering formula for the rotational loading of structures, it has been assumed $V_A=V_x=V_y$. The reason of such a simplification will be discussed in future in another authors' paper. However, it should be mentioned that the assumption of bidirectional phase velocity is considered against the current classical assumption of radial wave propagation (point seismic source case).

Noting that the torsional component about z-axis is independent of x- and y-axes the approximate relationship between the spectral density function (SDF) of rotational, $S_\theta^g(\omega)$, and translational accelerations can be obtained as:

$$\left\{S_\theta^g(\omega)\right\} = \left\{\omega^2 \frac{S_{uz}(\omega)}{V_A^2}; \omega^2 \frac{S_{uz}(\omega)}{V_A^2}; \frac{\omega^2}{4V_A^2}\left(S_{ux}(\omega) + S_{uy}(\omega)\right)\right\} \quad (7)$$

where $S_{ux}(\omega)$, $S_{uy}(\omega)$, and $S_{uz}(\omega)$ are, respectively, the SDF of the translational accelerations along principal axes. Considering the fact that random vibration-based response spectra of ground motions can be written as:

$$\mathrm{SD}^2 = C_d^2 \int_{-\infty}^{\infty} S(\omega)|H(\omega)|^2 d\omega \quad (8.a)$$

$$\mathrm{SV}^2 = C_v^2 \int_{-\infty}^{\infty} \omega^2 S(\omega)|H(\omega)|^2 d\omega \quad (8.b)$$

where $H(\omega)$ is the frequency response function, SD and SV are respectively displacement and velocity response spectrum, and also C_d and C_v are their corresponding peak factors. Moreover, assuming that the $S_{ux}(\omega)$ and $S_{uy}(\omega)$ are equal, the approximate forms of the acceleration response spectra of the rotational components, SA_θ, can be obtained by substituting Eq. 7 into Eq. 8.a and considering Eq. 8.b as follows:

$$\mathrm{SA}_{\theta x} = \mathrm{SA}_{\theta y} = \frac{2\pi \cdot \mathrm{SA}_{uz}}{T_\theta \cdot V_A}, \mathrm{SA}_{\theta z} = \frac{\sqrt{2}\pi \cdot \mathrm{SA}_{ux}}{T_\theta \cdot V_A} \quad (9)$$

in which SA_{ux} and SA_{uz} are the acceleration response spectra of the translational components along x- and z-axes, and T_θ is fundamental period of rotational single degree of freedom system. However, the acceleration response spectrum usually decrease with increase of the period of system vibration, but this reduction according to Eqs. 8.a and 8.b is more pronounced for rotational components than translational ones.

Figure 1 shows the normalized acceleration response spectra of translational and rotational components corresponding to a local earthquake recorded at HACC station of HGS-array with six components (Taiwan, 2007). Here, it should be noted that Fig. 1 shows the mean spectra of horizontal and rocking components of this event. As it can be seen, rotational acceleration response spectra rapidly decrease with increasing structural period; and the effects of rotational components are negligible in the long periods of vibration. From here, it can be inferred that the rotational acceleration components can only be destructive in seismic loading of structures which are stiff (short-period) like nuclear reactors, or sensitive to the high-frequency motions such as irregular structures or secondary systems. For instance, the high-frequency content of rocking components may severely increase the contribution of the specific higher modes of vibration to structural responses of vertically irregular structures and cause local structural and non-structural damages during SGMs.

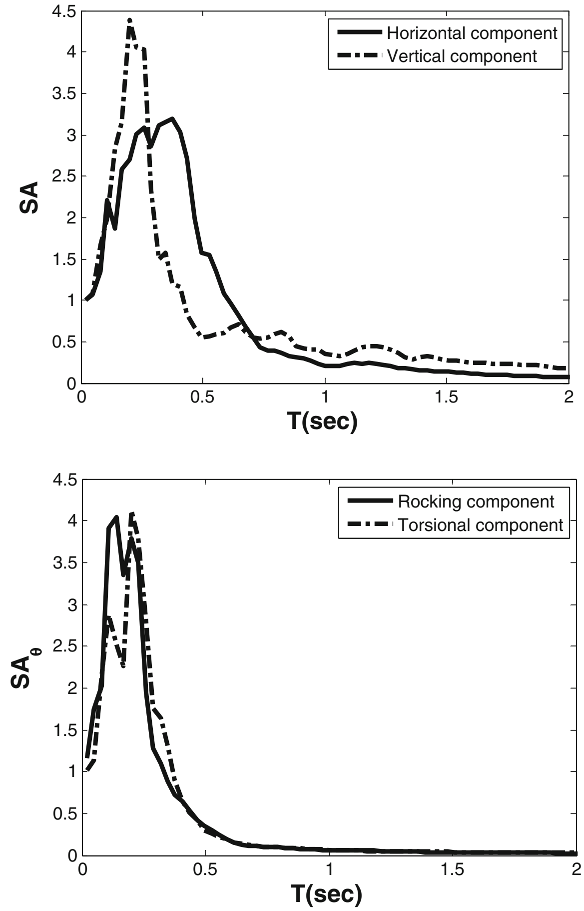

Fig. 1 Acceleration response spectra of translational and rotational components for damping ratio of 0.05

Herein, the dynamic analysis of the seismic behavior of the structures subjected to the rotational motions is performed using the random vibration theory. Also, to model the SDF of the input translational acceleration, the filtered Kanai-Tajimi spectrum (Ruiz and Penzien 1969), is used:

$$\frac{S(\omega)}{S_0} = \frac{1 + 4\xi_s^2(\omega/\omega_s)^2}{\left[1-(\omega/\omega_s)^2\right]^2 + 4\xi_s^2(\omega/\omega_s)^2} \times \frac{(\omega/\omega_g)^4}{\left[1-(\omega/\omega_g)^2\right]^2 + 4\xi_g^2(\omega/\omega_g)^2} \times \exp\left(-\frac{(\omega/\omega_s)^2}{2(1+\xi_s)^2}\right) \tag{10}$$

where ω_g, ω_s, ξ_g, ξ_s, and S_0 are the empirical parameter determined by fitting Eq. 10 to the SDF of the recorded accelerograms. Table 1 shows the considered values of these empirical parameters in this study. Using Eq. 10, the SDF of the horizontal and vertical acceleration components have been drawn in Fig. 2. Also, considering Eq. 7, the SDF of the rotational acceleration components for a medium soil are obtained using $V_A = 0.6 Km/s$ and their frequency contents have been shown in Fig. 3.

3 Rocking component in structural loading

In this section, the influences of the rocking components on the lateral force of ordinary buildings are studied. It has been assumed that (1) supporting columns rest directly on the soil without any mat foundation and the dynamic soil structure interaction effects is neglected; (2) responses are small and remain in the linear elastic range.

3.1 Contribution of rocking component to lateral earthquake force

The differential equations of motion for a shear-type building model with N-degrees of freedom subjected

Table 1 Parameters of filtered Kanai-Tajimi spectrum

SDF	S_0	ξ_g	ω_g (rad/s)	ξ_s	ω_s (rad/s)
z	0.4	0.6	1	0.4	10
x	1	0.6	0.75	0.3	7.5

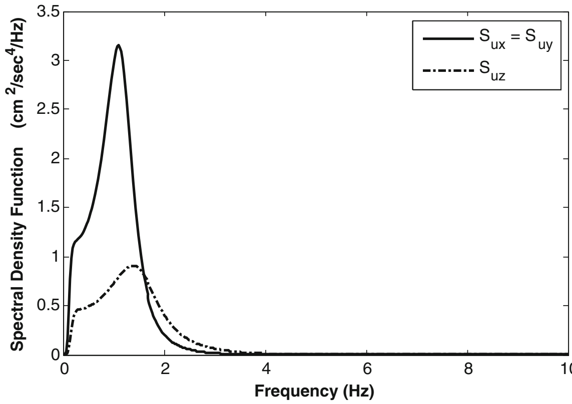

Fig. 2 The SDF of the translational acceleration components given by Eq. 10

to the base excitation of the rocking acceleration component, $\ddot{\theta}_{gy}$, can be expressed as:

$$[M]\cdot\left\{\ddot{U}\right\} + [C]\cdot\{\dot{U}\} + [K]\cdot\{U\} = -[M]\cdot\{h\}\cdot\ddot{\theta}_{gy} \tag{11}$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices, $\{h\}$ is the influence vector, $\{U(t)\}=\{\varphi\}[q(t)]$ is the displacement vector of structure, and $\{\varphi\}$ is the mode shape vector. In the case of proportionally damped matrix, the jth modal coordinate, q_j, is:

$$\ddot{q}_j(t) + 2\xi_j\omega_j\dot{q}_j(t) + \omega_j^2 q_j(t) = \gamma'_{\mathrm{j}}\ddot{\theta}_{gy} \tag{12}$$

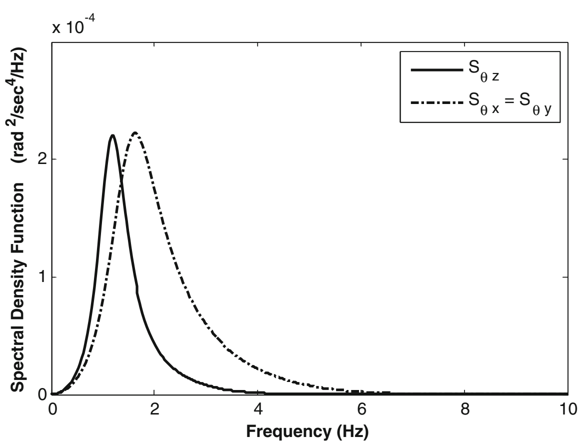

Fig. 3 The SDF of the rotational acceleration components given by Eq. 7

where ω_j and ξ_j are the modal frequency and damping ratio, $\gamma'_{\mathrm{j}} = -\left(1 \big/ M_j^m\right) \sum_{i=1}^{N} m_i \phi_{ij} h_i$, h_i and m_i are the height and mass of ith floor above the base, and M_j^m is the jth modal mass. For such a system, the jth modal response becomes:

$$q_j(t) = \gamma_j H_j \int_0^t \ddot{\theta}_{gy}(\tau) h_j(t-\tau) d\tau \tag{13}$$

where

$$H_j = \frac{\sum_{i=1}^{N} m_i \phi_{ij} h_i}{\sum_{i=1}^{N} m_i \phi_{ij}}, \gamma_j = \frac{-1}{M_j^m} \sum_{i=1}^{N} m_i \phi_{ij} \tag{14}$$

and $h_j(t\text{-}\tau)$ is the unit impulse response function of j^{th} mode. Also, the story shear can be obtained by:

$$\{f_s(t)\} = [K]\{U(t)\} = \sum_{j=1}^{N} \omega_j^2 [M] \{\phi_j\} q_j(t) \tag{15}$$

Using Eq. 15, the induced base shear due to the excitation of the rocking component becomes:

$$V_{\theta y}(t) = \sum_{i=1}^{N} f_{si} = \sum_{j=1}^{N} \gamma_j \omega_j^2 M_j^m q_j(t) \tag{16}$$

In special case, when the first mode shape can be assumed linear ($\phi_{i1} = h_i/H$), using the orthogonality property of modes and note to $\sum_{i=} 1^N m_i \phi_{ij} h_i = H\{\phi_1\}^T M\{\phi_j\}$, it can be shown that for all j≠1, the effective modal heights, H_j, are zero. In this case, Eq. 16 becomes:

$$V_{\theta y}(t) = \gamma_1^2 \omega_1^2 M_1^m H_1 \int_0^t \ddot{\theta}_{gy}(\tau) h_j(t-\tau) d\tau \tag{17}$$

From Eq. 17, the maximum base shear due to the rocking component by using Eq. 9 can be written as:

$$V_{\theta y-\max} = \frac{2\pi \overline{M}_1 H_1}{T_1 V_A} SPA_w(\omega_1, \xi_1) \tag{18}$$

where SPA is the pseudo-acceleration response spectrum and $\overline{M}_1 = \gamma_1^2 M_1^m$ is the effective mass of the first mode of the structural vibration. On the other hand, the base shear due to the excitation of the horizontal acceleration can similarly be expressed as:

$$V_u(t) = \sum_{j=1}^{N} \gamma_j^2 \omega_j^2 M_j^m \int_0^t \ddot{u}_g(\tau) h_j(t-\tau) d\tau \tag{19}$$

Similarly the maximum base shear due to the horizontal component can be written as:

$$V_{u-\max} = \eta \overline{M}_1 SPA_u(\omega_1, \xi_1) \tag{20}$$

where η is a non-dimensional factor with value in the range between 1 and 1.5 for typical multi-story buildings. Defining $\rho_{wu} \equiv SPA_w/SPA_u$ and assuming λ as the cross-correlation coefficient of the rocking and horizontal motions, the maximum base shear due to the combined action of the horizontal and rocking components can be written as:

$$V_{u\theta y-\max} = \frac{\mathrm{SPA}_u(T_1, \xi_1)}{g} \sqrt{(1+\theta_1)^2 + 2(\lambda - 1)\theta_1} \cdot \overline{W} \tag{21}$$

where g is the gravitational acceleration, $\overline{W} = \eta \overline{M}_1 g$ is the effective weight of the structure vibration, and $\theta_1 = (2\pi \rho_{wu} H_1)/(T_1 V_A \eta)$ in which $H_1 = \sum_{i=1}^{N} m_i h_i^2 \Big/ \sum_{i=1}^{N} m_i h_i$. The H_1 for a N-story building with regular configuration in elevation, can be obtained by $(2N+1)H/3N$. Two conservative assumptions are necessary to obtain a practical formula for the base shear, V, in seismic design codes due to the combined action of the horizontal and rocking components: (1) considering a linear fundamental mode shape for low-to-moderate rise buildings; (2) assuming that the horizontal and rocking components are perfectly correlated. In this case, code base shear can be modified using Eq. 21:

$$V_{nu} = V(1+\theta_1) \tag{22}$$

To examine the applicability of the Eq. 22, at first, the variations of ρ_{wu} as a function of period for the assumed SDFs is calculated (see Fig. 4). Next, the variations of parameter θ_1 versus vibration periods is compared with the exact ratio of V_{nu}/V calculated from the dynamic analysis of a single degree of freedom system under considered SDFs and it is shown in Fig. 5. It can be seen that the proposed formula can conservatively evaluate the seismic loading of structures subjected to the rocking component. It can also be observed that for tall and short period structures, such as nuclear reactors, the seismic loading of

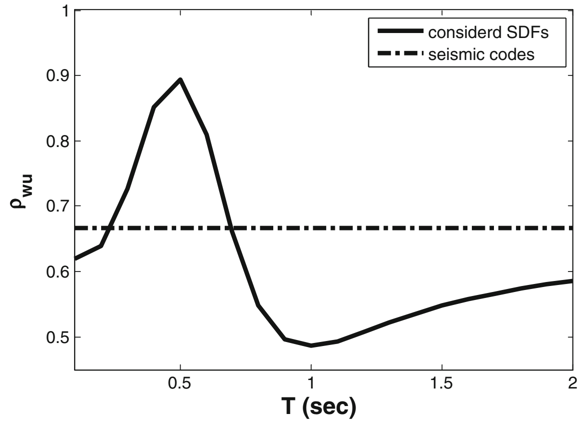

Fig. 4 Variations of ρ_{wu} versus period

rocking components can be as large as translational ones or even larger. This phenomenon occurs because of the differences in the frequency content between rotational and translational components, and it can seriously affect seismic behavior of structures located on soft soil.

For engineering application, in order to use Eq. 22 in seismic design codes, it may be assumed that the apparent velocity is conservatively equal to the average shear wave velocity (V_s) in the top 30 m of the ground profile, $\rho_{wu} \cong 2/3$, and $\eta \cong 1$. In this case, θ_1 can approximately be expressed as:

$$\theta_1 \cong \frac{4.2H_1}{TV_S} \tag{23}$$

where T is the fundamental period of the structure.

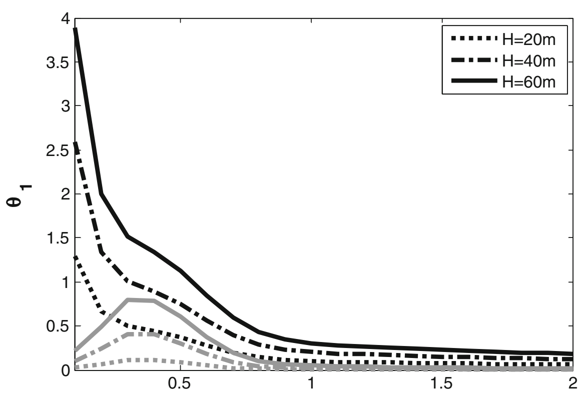

Fig. 5 Variations of θ_1 calculated from proposed formula (*black lines*) and from dynamic analysis (*gray lines*) versus the structural periods of a single degree of freedom system for H=20, 40, and 60 m

4 Torsional component in structural loading

The concept of center of rigidity (CR) arises from single-story structures with rigid floor diaphragm where there is always a point on the floor (CR) which, if a static load (of arbitrary magnitude and direction) is applied through this point, will translate the floor without rotation. This concept cannot always be extended to multi-story structures in terms of a set of points at the floor levels that possess the same property. However, there is a very special class of multistory buildings, namely buildings having vertical resisting elements with proportional stiffness matrices, in which a set of the CRs can be defined in the aforementioned strict sense and lie on a common vertical line (Riddel and Vasquez 1984; Tso 1990). Since seismic provisions are usually based on the studies concerning the torsional response of single-story systems and dynamic response of plane frames (Kan and Chopra 1977; Tso and Dempsy 1980; Hejal and Chopra 1989; Ghafory-Ashtiany 2001); therefore, these provisions rigorously apply to the uniform multistory shear or flexural type structures (proportionate buildings).

Thus, in this section, a new formula for the inclusion of the loading effects of rotational ground motions on the accidental eccentricity corresponding to the proportionate buildings is presented. In this case, because of the relationship between the base shear and accidental eccentricity in seismic codes, it is necessary to consider combined action of horizontal, rocking and torsional on structural loading. To achieve this, at first, in order to give a basic insight into torsional loading, the accidental eccentricity due only to torsional loading is presented in a static form. Next, a relation to evaluate the equivalent accidental eccentricity is derived using the numerical results obtained from the dynamic analyses.

4.1 Static accidental eccentricity for a single-story building

Consider a linear single-story building with orthogonal arrangement of lateral-load resisting system connected by rigid floor diaphragm as shown in Fig. 8. For lateral force analysis in the x-direction, the building plan is treated as symmetric about the y-axis without loss of generality because building codes need such independent analyses in the x- and y-directions.

In order to determine the static accidental eccentricity, e_a^{static}, with no important loss of generality, the UBC 1997 values $\alpha = \delta = 1$ is chosen in Eqs. 2.a and 2.b. The static accidental eccentricity is introduced as ratio of the induced maximum torsional moment due only to the earthquake torsional component, θ_{gz}, to lateral force due to the simultaneous excitations of the earthquake rocking and horizontal components as follows:

$$e_a^{\text{static}} = \frac{M^t - V_{\text{nu}}e}{V_{\text{nu}}} = \frac{K_\theta}{K} \cdot \frac{\theta_{\max}}{u_{\max}} \tag{24}$$

in which M^t is the total torque induced in the system due to seismic loading of ground motions, K and K_θ are the lateral and torsional stiffness of the structure, $\theta_{\max}$ is the peak rotational displacement due only to the torsional excitation. The normalized static accidental eccentricity is given using Eqs. 9 and 22:

$$e_{ab}^{\text{static}} = \frac{e_a^{\text{static}}}{b} = \frac{\sqrt{2}K_\theta \omega_\theta}{2bKV_A(1+\theta_1)} \cdot \frac{\text{SD}_u(\omega_\theta, \xi)}{\text{SD}_u(\omega_1, \xi)} \tag{25}$$

where b is the plan dimension of the structure normal to the considered direction of ground motion. Defining $\Omega = \omega_\theta/\omega_1 = K_\theta/r^2K$ as the ratio between the uncoupled torsional frequency and the lateral frequency of the structure, assuming $V_A \cong V_S$ and using Eq. 23, then Eq. 25 can be simplified to:

$$e_{ab}^{\text{static}} \cong \frac{\sqrt{2}\pi \cdot r^2 \Omega}{b(4.2H_1 + TV_S)} \cdot \frac{\text{SPA}_u(\omega_\theta, \xi)}{\text{SPA}_u(\omega_1, \xi)} \tag{26}$$

where the radius of gyration about CR is $r = \sqrt{(b^2+a^2)/12} = b\sqrt{(1+\varepsilon^2)/12}$ in which $\varepsilon = a/b$ is the plan aspect ratio, a is the side dimension parallel to the lateral seismic loading, and the SPA is the pseudo-acceleration response spectrum. From Eq. 26, it can be deduced that the main effective structural parameters in the torsional loading of an equivalent linear symmetrical one-story building are H_1, T, b, ε, and Ω.

4.2 Equivalent accidental eccentricity for single-story buildings

In order to determine the equivalent accidental eccentricity, e_a, consider the linear single-story building shown in Fig. 6. The modified base shear, V_{nu}, is applied at the distance e from the CR produces a rotational displacement equal to θ. The maximum dynamic rotational displacement due only to the rocking and horizontal components of ground motion (β=0) can be written as:

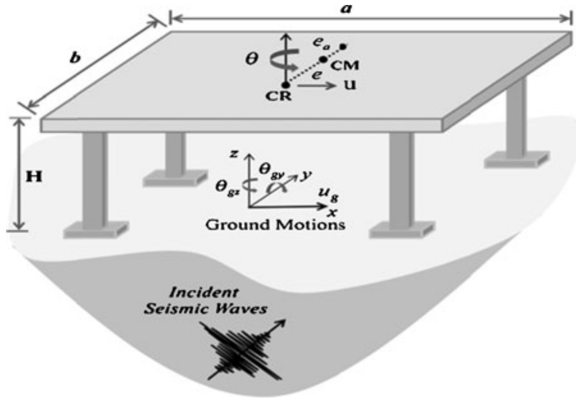

Fig. 6 Considered model for an unbalance torsionally structure

$$\theta = \frac{V_{nu}e}{K_\theta} \tag{27}$$

By applying the same static force, V_{nu}, at the distance e+e_a relative to the CR, where e_a=βb; the maximum dynamic rotational displacement of the system with accidental eccentricity (considering the effects of the torsional motion), $\widehat{\theta}$, becomes:

$$\widehat{\theta} = \frac{V_{nu}(e+e_a)}{K_\theta} \tag{28}$$

Solving Eqs. 27 and 28 for e_a and considering u=V_{nu}/K as the peak dynamic translational displacement at the floor CR, we obtain:

$$e_{ab} = \frac{e_a}{b} = \frac{\left(\widehat{\theta} - \theta\right)}{ub}\Omega^2 r^2 \tag{29}$$

Using Eq. 29, the general trends of the equivalent static accidental eccentricity calculated for the various sets of the linear structures with different dynamic properties under varied excitations showed that the equivalent accidental eccentricity for the proportionate buildings can approximately be evaluated by:

$$e_{ab}^* = \frac{e_a^*}{b} = \begin{cases} \frac{4.5\Omega r^2}{b(2H_1+TV_S)} & , \quad T \geq 0.4S \\ \frac{2.25\Omega r^2}{b(H_1+0.2V_S)} & , \quad T \leq 0.4S \end{cases} \tag{30}$$

in which T is the lateral fundamental period of structure for while the structure is considered uncoupled. In continuation, the applicability of Eq. 30 in estimating the equivalent accidental eccentricity is examined. To achieve this, the equivalent accidental eccentricity for different values of the effective structural parameters is calculated by considering the system shown in Fig. 6. The influences of the effective structural parameters on the accidental eccentricity are shown in Figs. 7–11.

As it can be seen in Fig. 7, with the increase of the plan dimension and lateral fundamental frequency of the structure the accidental eccentricity will increase. Also, the equivalent accidental eccentricity decrease with the increase of the static eccentricity. In general, the accidental eccentricity arrives at its maximum value for symmetric structures. Figure 8 shows that for a constant value of b, the decrease in the plan aspect ratio of structure, ε, leads to the decrease of the accidental eccentricity. In fact, since the reduction in the ratio a/b results in decreasing the ratio r/b; therefore, according to Eq. 26, it can easily be inferred why this causes that the accidental eccentricity reduces.

Figures 9 and 10 show the effects of the frequency ratio Ω on the value of the accidental eccentricity. Two points can be drawn from these figures: First, the increase in e_{ab} is largest for symmetric torsionally stiff structures. Also, the increase in accidental eccentricity calculated by dynamic analysis for the symmetric system with $\Omega \prec 1$ tends to be smaller than that predicted by proposed formula given in Eq. 30, and vise versa for $\Omega \succ 1$ and $T \succ 0.9$sec. Second, for asymmetric structures the effects of the static eccentricity on e_{ab} decrease with the increase of period and decrease of frequency ratio. This variation of Ω and its influence on the value of the accidental eccentricity implies to an important property of torque loading due to the torsional component. Indeed, a common assumption in seismic design of structures is that the effects of the lateral–torsional coupling on the seismic response of in-plan irregular systems decrease as increasing Ω, but here, it has been shown that increase in the value of Ω can conversely increase torque loading of torsional components. Thus, it should be kept in mind that increase of Ω does not generally result in reducing the torsional effects in seismic behavior of structures. Apparently, for a deeper insight into this area requires more research.

As shown in Fig. 11, when the height of structure increases the rotational loading due to the rocking motions can cause the accidental eccentricity decreases. Indeed, when the total lateral loading force of structure increases, the simultaneous seismic

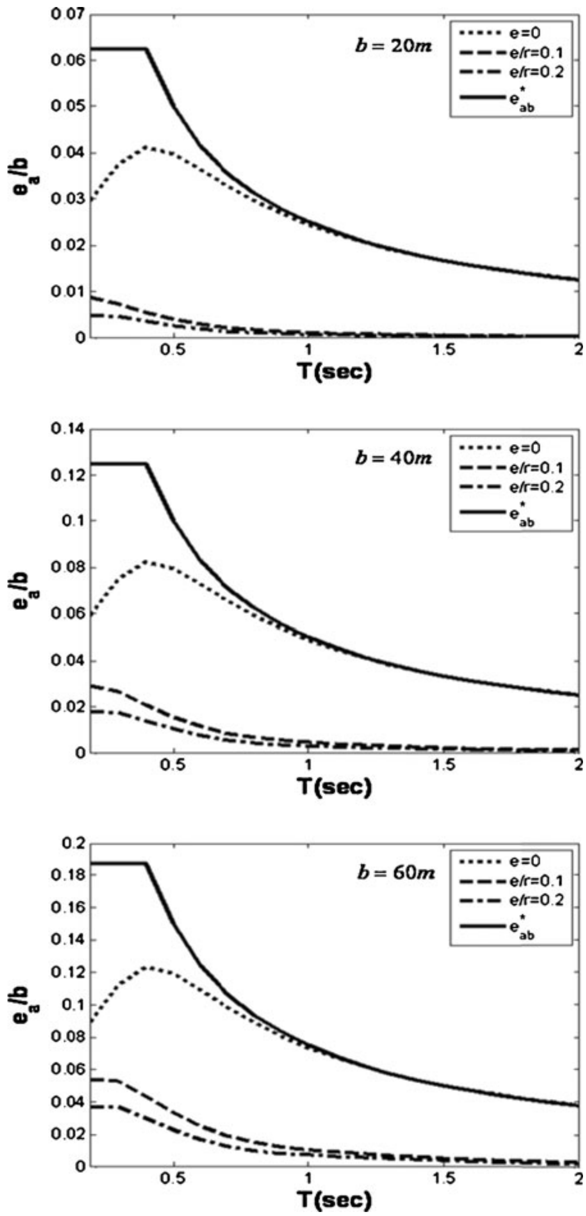

Fig. 7 Variation of accidental eccentricity as a function of period for structure with square plan and for effective parameters of e/r=0, 0.1, 0.2, Ω=1, H=0, and b=20, 40, 60m

loading of the rocking and translational motions cause that a smaller accidental eccentricity is needed to produce a constant torsional moment in structure.

Although proposed formula underestimates the accidental eccentricity for symmetric tall structures which are torsionally stiff, relative to the numerical results of dynamic analysis but this difference between the results is negligible. The above observations indicate that the proposed formula can efficiently estimate

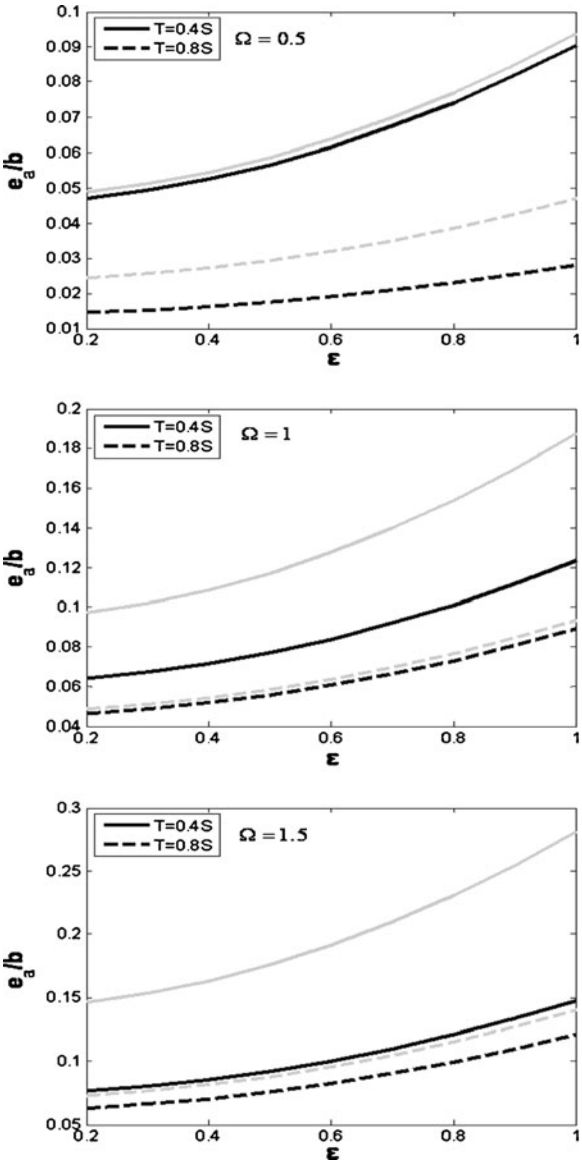

Fig. 8 Variation of accidental eccentricity (*gray line*: e^*_{ab}, *black line*: e_{ab}) versus $\varepsilon=a/b$, for structure with b=60 m and for effective parameters of e/r=0, T=0.4, 0.8 s of, H=0, and Ω= 0.5, 1, 1.5

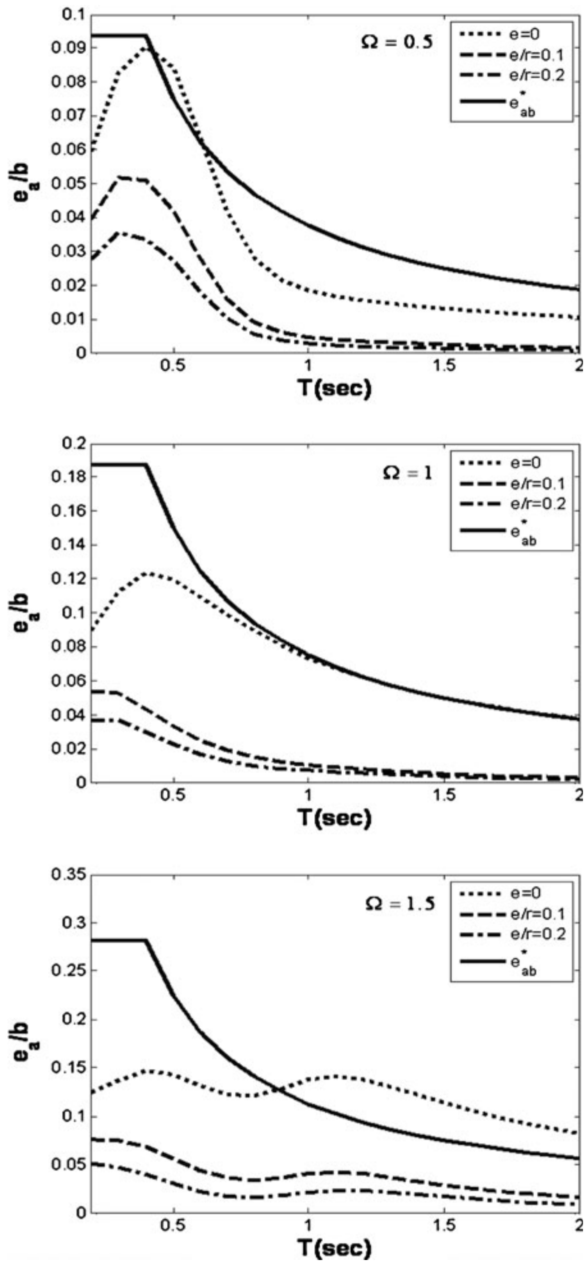

Fig. 9 Variation of accidental eccentricity as a function of period for structure with b=60 m and for effective parameters of e/r=0, 0.1, 0.2, ε=1, H=0, and Ω=0.5, 1, 1.5

the maximum values of the normalized equivalent accidental eccentricity, e_{ab}, for the type of the structural systems that have considered in this study. It should also be noted that the accidental eccentricity specification of β=0.05 is not an acceptable estimation for wide symmetric multi-story buildings which having short fundamental period and large value of Ω. However, since the symmetric structures are very rare and even in such systems, an uneven distribution of mass (for example live load) may cause eccentricity; therefore, the value of β=0.05 in seismic codes may be a fairly good approximation of the maximum accidental eccentricity in common structures due only to the earthquake torsional component. To verify this, further research and study on multi-story buildings with modern structural designs are needed.

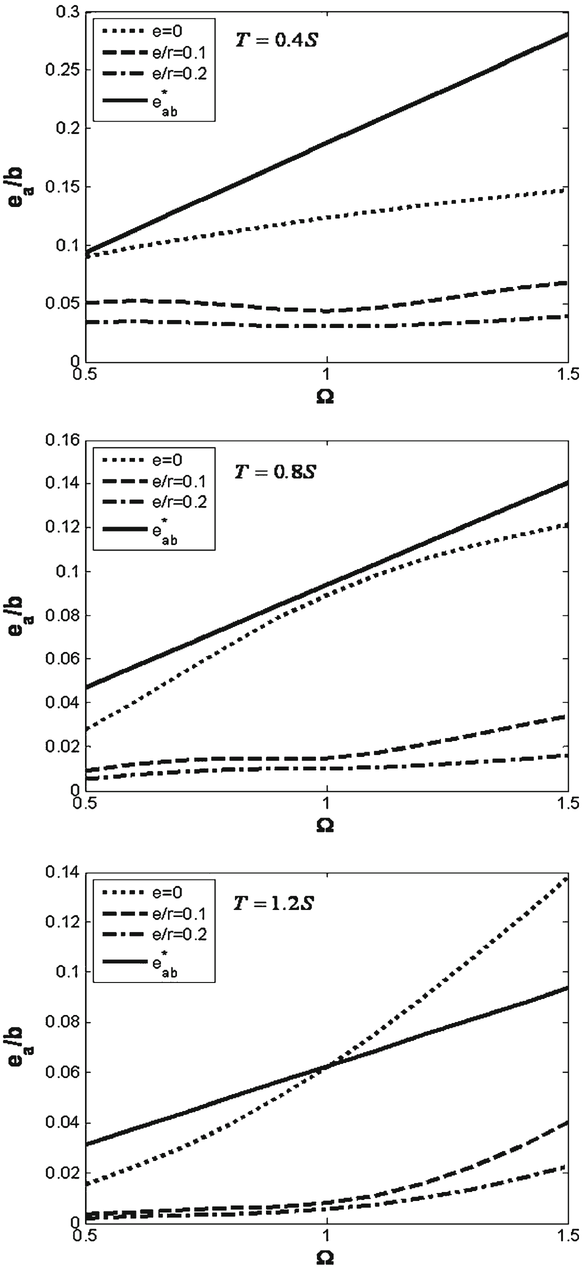

Fig. 10 Variation of accidental eccentricity as a function of Ω, for structure with b=60 m and for effective parameters of e/r=0, 0.1, 0.2, ε=1, H=0, and T=0.4, 0.8, 1.2 s

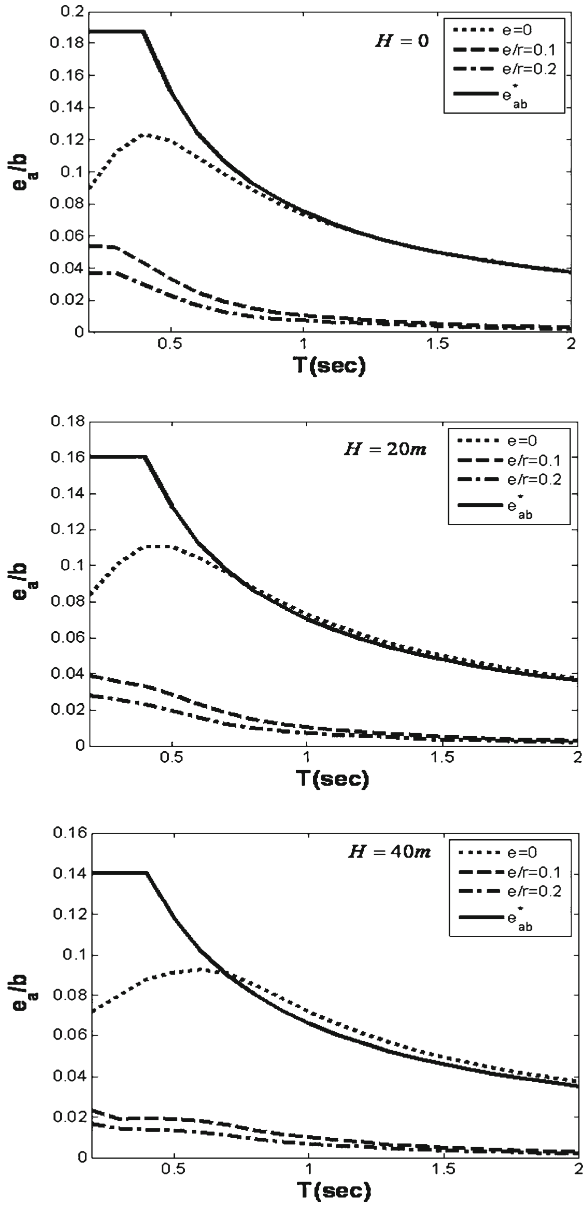

Fig. 11 Variation of accidental eccentricity as a function of period for structure with b=60 m and for effective structural parameters of e/r=0, 0.1, 0.2, Ω=1, ε=1, and H=0, 20, 40m

5 Limitations

The relationships developed in this study are based on the some major limitations as follows:

First, the effects of spatial variation of body waves have only been considered to present formulas for the rotational loading of structures. It should be noted that there are other phenomena that can lead to the rotational excitations of structures such as surface waves, special rotational waves, block rotation, topographic effects, and soil structure interaction.

Second, a single apparent velocity has been assumed to take into account the wave passage effects. Although, authors are aware that this assumption ignores the dispersion of SGM waves, but we believe that its use is justified as a first step in the development of simple quantitative criteria for engineering applications.

Third, the effect of the phase-delay between earthquake rotational and translational components on the seismic loading of structures has conservatively been ignored. It should be kept in mind that in real conditions, due to the phase-shift and interaction between the rotational and translational components, the effects of rotational components may be beneficial on the seismic loading of the structures and lead to the reduction of structural responses.

Finally, the new formulas proposed herein does not account for inelastic actions of the structures. Although, this approach has the advantage of clear physical interpretation of seismic behavior of structures subjected to rotational excitation but the conclusions drawn are only applicable to the engineering structures which remain elastic during small and moderate earthquakes.

6 Conclusions

The new formulas for inclusion of the earthquake rotational motion effects in the seismic loading of low-rise multi-story buildings have been presented. The effective structural parameters in the rotational loading of such structures have been determined and their influences on the linear dynamic behavior of structures have been studied. The following conclusions based on the trends of the numerical results obtained and under the assumptions of this study may be drawn:

- The acceleration response spectra of rotational components tend to decay faster than corresponding translational ones as periods of vibration increase. Thus, the seismic loading of these motions on the overall behavior of long-period structures ($T \geq 2$ s) may be ignored.
- The rotational acceleration components are of more high frequency than corresponding translational ones. Thus, they may remarkably change seismic loading of the structures which are sensitive to the high-frequency motions, such as the secondary systems. The contribution of these components to the seismic excitation of tall short-period structures like nuclear reactors can be as large as translational ones or even larger.
- The effects of the torsional acceleration component on structural loading tend to increase with the increase of Ω and the radius of gyration of the building floors. In contrast, it decreases by increasing the static eccentricity and fundamental lateral period of building.
- In asymmetric structures, with the increase of period and the decrease of frequency ratio Ω, the influences of static eccentricity on e_{ab} decrease and it approaches to a small constant value. Besides, the effects of torsional motions can be ignored for tall multi-story buildings which have strong asymmetry in plan.
- The value of the eccentricity $0.05b$ which prescribed in most of the current seismic design codes for the accidental torsional effects is mostly a conservative approximation for accidental eccentricity due only to the influences of the torsional component in asymmetric buildings.

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